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2002 J. Phys.: Condens. Matter 14 7471

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# Strong-coupling effects in ‘dirty’ superfluid $^3\text{He}$

G Baramidze and G Kharadze

Andronikashvili Institute of Physics, Georgian Academy of Sciences, 6 Tamarashvili St., 380077, Tbilisi, Georgia

E-mail: gogi@iph.hepi.edu.ge

Received 23 November 2001, in final form 5 July 2002

Published 2 August 2002

Online at [stacks.iop.org/JPhysCM/14/7471](http://stacks.iop.org/JPhysCM/14/7471)

## Abstract

The contribution of the strong-coupling effects to the free energy of the ‘dirty’ superfluid  $^3\text{He}$  is estimated using a simple model. It is shown that the strong-coupling effects are less susceptible to quasiparticle scattering events in comparison to the weak-coupling counterpart. This supports the conclusion of stabilization of the B-phase in the aerogel environment at pressures where the A-phase is favoured in bulk superfluid  $^3\text{He}$ , in accordance with recent experimental observations in zero magnetic field.

## 1. Introduction

One of the actively investigated problems in low-temperature physics is the search for the properties of a ‘dirty’ superfluid Fermi system, such as liquid  $^3\text{He}$ , confined to a high-porosity aerogel environment. A number of experimental observations [1–10] revealed new aspects of the behaviour of the ordered state of the superfluid  $^3\text{He}$  in the presence of quasiparticle scattering against a random system of silica strands forming the skeleton of the aerogel.

The theoretical attempts to interpret these experiments, although partly successful, are still insufficient to describe the main body of accumulated information and the details of the phase diagram, in particular. The theoretical approach adopted up to now is based on a weak-coupling approximation. One of the conclusions obtained in this way is the claim that in zero magnetic field the quasiparticle scattering on the spatial irregularities (‘impurities’) promotes the stability of the isotropic B-phase in the domain of the  $P$ – $T$  phase diagram where in bulk (‘pure’) superfluid  $^3\text{He}$  the anisotropic A-phase is favoured [11, 12]. This theoretical result means that at pressures above the polycritical value  $P_{c0}$  the ‘dirty’ B-phase overcomes the so-called strong-coupling effects and should appear as an equilibrium superfluid state of liquid  $^3\text{He}$  confined to the aerogel environment. This conclusion is based on a supposition that the strong-coupling effects (which are also subject to ‘impurity’ renormalization) are less susceptible to the quasiparticle scattering events.

In what follows we explore this question in some detail. It will be shown (using a simple model) that it does indeed seem likely that, although the strong-coupling effects are enhanced

due to the finite value of the quasiparticle mean free path, the B-phase is still able to prevail over the A-phase at high pressures. Quite recently, using a high-frequency acoustic technique [13], the 3D phase diagram in  $(P, T, B)$  space was constructed for superfluid  $^3\text{He}$  confined to 98% porosity aerogel (see also [14]). The pressure range covered extended from 15 bar up to the melting pressure. One of the most striking observations is that in zero magnetic field ( $B = 0$ ) and at all pressures above 15 bar the phase transition to the B-like phase occurs (at  $T_c(P)$  with no signs of the polycritical point (PCP) at which the A- and B-phases meet in bulk superfluid  $^3\text{He}$  at  $P_{c0} = 21$  bar). It appears that the PCP for  $^3\text{He}$  in 98% porosity aerogel is absent because  $P_c$  is pushed to above the solidification pressure, and thus is unobservable. The results of our theoretical consideration seem to be in accordance with the above-mentioned experimental observations.

## 2. Strong-coupling effects in ‘dirty’ superfluid $^3\text{He}$

The weak-coupling approach in treating the properties of superfluid phases of liquid  $^3\text{He}$  disregards the inverse action of the ordering on the pairing interaction (so-called strong-coupling effects). The importance of this feedback effect is well known [15] and is mainly due to an attractive contribution of the spin excitations in the strongly correlated system to the effective quasiparticle interaction. Physically the strong-coupling feedback effect stems from the fact that the spin susceptibility of liquid  $^3\text{He}$  is sensitive to the character of the spin-triplet Cooper pairing order parameter. In what follows, using a simple model, we are going to estimate the influence of the quasiparticle scattering on the strong-coupling effects having in mind an application to the superfluid  $^3\text{He}$  filling the low-density aerogel. It should be stressed that a much more refined approach in treating the strong-coupling effects is based on a systematic expansion of the free energy of superfluid  $^3\text{He}$  in powers of  $T_c/T_F$ . This programme has been realized in [16] (see also the review article [17] and [18]). This approach captures the contributions to the strong-coupling effects stemming not only from the spin fluctuations but from the transverse current fluctuations as well. Unfortunately, it is an extremely difficult task to treat the ‘impurity’ effects even within relatively simple dynamical spin-fluctuation model used in [19], to say nothing of the more sophisticated approach mentioned above. Instead, we will rely on a static approximation [20] which disregards the retardation effects in treating an attractive interaction between quasiparticles via the exchange of the ‘paramagnons’.

We start from the momentum-space Fourier component of the static spin-susceptibility tensor

$$\chi_{\mu\nu}(\vec{q}) = -\frac{1}{2}T \sum_{\omega} \sum_k \text{Tr}[\check{G}_{\omega}(\vec{k})\check{\sigma}_{\mu}\check{G}_{\omega}(\vec{k} + \vec{q})\check{\sigma}_{\nu} - \check{F}_{\omega}(\vec{k})\check{\sigma}_{\mu}\check{F}_{\omega}(\vec{k} + \vec{q})\check{\sigma}_{\nu}^{\prime r}], \quad (1)$$

where  $2 \times 2$  spin matrices  $\check{G}_{\omega}(\vec{k})$  and  $\check{F}_{\omega}(\vec{k})$  denote the Gorkov Green functions (in the Matsubara representation), describing an ordered (superfluid) Fermi system. Equation (1) can be used to construct an effective spin-dependent part of the interaction potential acting between quasiparticles with the matrix elements

$$J_{\mu\nu}(\vec{k}, \vec{k}')(\check{\sigma}_{\mu})_{\alpha\alpha'}(\check{\sigma}_{\nu})_{\beta\beta'}, \quad (2)$$

where

$$J_{\mu\nu}(\vec{k}, \vec{k}') = -I^2 \chi_{\mu\nu}(\vec{k} - \vec{k}') \quad (3)$$

with  $I$  standing for the local repulsive potential describing correlation effects.

In the random-phase approximation the susceptibility tensor  $\check{\chi}$  reads as

$$\check{\chi} = \check{\chi}^{(0)} + \check{\chi}^{(0)} I \check{\chi} = (\check{1} - I \check{\chi}^{(0)})^{-1} \check{\chi}^{(0)}, \quad (4)$$

where  $\check{\chi}^{(0)}$  stands for the spin susceptibility in the absence of correlation effects ( $I = 0$ ). In the vicinity of the critical temperature of the phase transition to the superfluid state,

$$\check{\chi}^{(0)} \simeq \chi_N^{(0)} \check{1} + \delta\check{\chi}^{(0)} \quad (5)$$

with  $\chi_N^{(0)}$  being the normal-state susceptibility. The superfluid contribution  $\delta\check{\chi}^{(0)}$  is quadratic in the order parameter of the superfluid state. Consequently, at  $T \lesssim T_c$ ,

$$J_{\mu\nu} \simeq -\frac{I^2 \chi_N^{(0)}}{1 - I \chi_N^{(0)}} \delta_{\mu\nu} - \left( \frac{I}{1 - I \chi_N^{(0)}} \right)^2 \delta\check{\chi}_{\mu\nu}^{(0)}. \quad (6)$$

The results concerning the strong-coupling contribution in  $^3\text{He}$  and based on the above-mentioned simple model are described in [21]. They show that this model reflects the essence of the feedback effects at least qualitatively. At the same time, in the framework of the static model adopted the technical side of the calculation of the quasiparticle scattering effects (which is our main goal) is simplified considerably. We hope that the static model, which we adopted, gives at least a qualitatively meaningful treatment of the *relative* stability of the A- and B-phases of a ‘dirty’ superfluid  $^3\text{He}$  confined to an aerogel environment in zero magnetic field.

In order to estimate the effects of the finite mean free path of the quasiparticles we address a self-consistency equation for the order parameter matrix  $\check{\Delta}$  which in the case of a spin-triplet Cooper pairing (appropriate to the superfluid  $^3\text{He}$ ) is given as

$$\check{\Delta} = \Delta_\mu (\check{\sigma}_\mu i\check{\sigma}_y). \quad (7)$$

The equation for the vector component  $\Delta_\mu$  reads as

$$\Delta_\mu(\hat{k}) = -T \sum_\omega \sum_{k'} V_{\mu\nu}(\vec{k}, \vec{k}') F_{\omega\nu}(\vec{k}'), \quad (8)$$

where  $\hat{k}$  is the unit vector along the momentum direction  $\vec{k}$ ,

$$V_{\mu\nu} = V \delta_{\mu\nu} + \delta V_{\mu\nu}, \quad (9)$$

and the feedback contribution  $\delta V_{\mu\nu} = J_{\lambda\lambda} \delta_{\mu\nu} - 2J_{\mu\nu}$ . In equation (8)  $F_{\omega\nu}$  denotes the  $\nu$ th vector component connected to  $\hat{F}_\omega$  in a similar way to in equation (7). In the model adopted,

$$\delta V_{\mu\nu} = -\left( \frac{I}{1 - I \chi_N^{(0)}} \right)^2 (\delta\chi_{\lambda\lambda}^{(0)} \delta_{\mu\nu} - 2\delta\chi_{\mu\nu}^{(0)}). \quad (10)$$

In terms of the quasiclassical function

$$f_{\omega\nu}(\hat{k}) = \frac{1}{\pi} \int_{-\alpha}^{+\alpha} d\xi F_{\omega\nu}(\hat{k}, \xi) \quad (11)$$

the self-consistency equation (8) reads as

$$\Delta_\mu(\hat{k}) = 2\pi T \sum_{\omega>0} \sum_{k'} \langle 3\hat{k}\hat{k}' g_{\mu\nu} f_{\omega\nu}(\hat{k}') \rangle, \quad (12)$$

where the brackets denote the averaging over the orientation of  $\hat{k}'$  and  $g_{\mu\nu}$  stands for an attractive component in the p-wave channel. According to our consideration  $g_{\mu\nu} = g\delta_{\mu\nu} + \delta g_{\mu\nu}$  with  $\delta g_{\mu\nu}$  stemming from the spin-dependent part of the quasiparticle interaction (see equation (10)).

For the case of bulk superfluid  $^3\text{He}$  near  $T_{c0}$  (the critical temperature of a pure system) and at  $q \ll k_F$ ,

$$\delta\chi_{\mu\nu}^{(0)}(\vec{q}) = -2\pi T \sum_{\omega>0} \frac{N_F}{2\omega^3} \left( \frac{2\omega}{qv_F} \right)^2 \left\langle \frac{\text{Re}(\Delta_\mu(\hat{k})\Delta_\nu^*(\hat{k}))}{(\hat{k}\hat{q})^2 + (2\omega/qv_F)^2} \right\rangle, \quad (13)$$

where  $N_F$  and  $v_F$  denote the density of states and the velocity of the quasiparticles at the Fermi level. After averaging over the orientation of  $\hat{q}$ , from equation (13) it is obtained that

$$\delta\chi_{\mu\nu}^{(0)}(q) = -\frac{N_F}{qv_F} \left[ 2\pi T \sum_{\omega>0} \frac{1}{\omega^2} \arctan(qv_F/2\omega) \right] \text{Re}\langle \Delta_\mu(\hat{k}) \Delta_\nu^*(\hat{k}) \rangle. \quad (14)$$

The main contribution to the feedback coupling constant  $\delta g_{\mu\nu}$  is to be extracted from the region of  $q \gg \xi_{c0}^{-1}$  where the coherence length  $\xi_{c0} = v_F/2\pi T_{c0}$ . In this limit, equation (14) gives

$$\delta\chi_{\mu\nu}^{(0)}(q) = -\frac{\pi}{2} \frac{N_F}{qv_F} \left( 2\pi T \sum_{\omega>0} \frac{1}{\omega^2} \right) \text{Re}\langle \Delta_\mu(\hat{k}) \Delta_\nu^*(\hat{k}) \rangle \quad (15)$$

and as a result

$$\delta g_{\mu\nu} = \frac{\delta g}{(\pi T_{c0})^2} \langle |\vec{\Delta}|^2 \delta_{\mu\nu} - \Delta_\mu \Delta_\nu^* - \Delta_\mu^* \Delta_\nu \rangle, \quad (16)$$

with

$$\delta g = \frac{1}{6} \left( \frac{\pi}{2} \right)^3 \left( \frac{IN_F}{1-IN_F} \right)^2 \frac{1}{k_F \xi_{c0}}. \quad (17)$$

Our main concern is to establish the modification of equation (16) caused by the quasiparticle scattering against the irregularities ('impurities') introduced by the presence of aerogel silica strands. The spin susceptibility is a two-particle correlator and the corresponding system of equations is to be addressed. The impurity scattering effects show up, in particular, as vertex corrections, complicating considerably the general consideration. Fortunately at  $\xi_c^{-1} \ll q \ll k_F$ , which is the region of the momentum transfer that we are interested in, the vertex corrections are small as long as  $(k_F \xi_c)^{-1} \ll 1$ . Finally, the essential contributions to the feedback effect modification due to the scattering events are simply realized by the substitution  $\omega \rightarrow \tilde{\omega} = \omega + \Gamma \text{sgn } \omega$  in equation (15), where  $\Gamma = \frac{c}{\pi N_F} \sin^2 \delta_0$  is the quasiparticle scattering rate (in what follows we adopt the so-called homogeneous scattering model (HSM) with the s-wave scattering channel only (see [11, 12]). Here  $c$  denotes the 'impurity' concentration and  $\delta_0$  is the phase shift at an s-wave scattering. As a result the coupling constant  $\delta g_{\mu\nu}$  (see equation (16)) is transformed to

$$\delta \tilde{g}_{\mu\nu} = \frac{\delta \tilde{g}}{(\pi T_c)^2} \langle |\vec{\Delta}|^2 \delta_{\mu\nu} - \Delta_\mu \Delta_\nu^* - \Delta_\mu^* \Delta_\nu \rangle, \quad (18)$$

where  $T_c$  stands for the critical temperature of the phase transition of liquid  $^3\text{He}$  in aerogel to an ordered state and

$$\delta \tilde{g} = \frac{1}{6} \left( \frac{\pi}{2} \right)^3 \left( \frac{IN_F}{1-IN_F} \right)^2 \frac{1}{k_F \xi_c} \frac{\psi^{(1)}(1/2+w)}{\pi^2/2}, \quad \xi_c = \frac{v_F}{2\pi T_c}. \quad (19)$$

Here  $\psi^{(m)}(z)$  denotes the poly-gamma function and  $w = \Gamma/2\pi T$ .

Taking into account that up to the third order in  $\vec{\Delta}$ , and in the presence of the quasiparticle scattering centres,

$$\vec{f}_\omega \simeq \frac{\vec{\Delta}}{|\tilde{\omega}|} - \frac{1}{2|\tilde{\omega}|^3} \left[ (\vec{\Delta}^* \vec{\Delta}) \vec{\Delta} + (\vec{\Delta}^* \times \vec{\Delta}) \times \vec{\Delta} - \frac{\Gamma \cos 2\delta_0}{|\tilde{\omega}|} (|\vec{\Delta}|^2 \vec{\Delta} + \langle (\vec{\Delta}^* \times \vec{\Delta}) \rangle \times \vec{\Delta}) \right], \quad (20)$$

after averaging over the orientation of  $\hat{k}'$  in the self-consistency equation (12), the equation for the order parameter  $\vec{\Delta}(\hat{k})$  near  $T_c$  reads (for the superfluid  $^3\text{He}$   $\Delta_\mu(\hat{k}) = A_{\mu i} \hat{k}_i$ )

$$\begin{aligned}
(a_1(T) - 1/g)\bar{\Delta}(\hat{k}) &= \frac{2}{3}a_3(-\frac{1}{2}\langle\bar{\Delta}^2\rangle\bar{\Delta}^*(\hat{k}) + \langle|\bar{\Delta}|^2\rangle\bar{\Delta}(\hat{k})) \\
&+ \langle\bar{\Delta}\Delta_v\rangle\Delta_v^*(\hat{k}) + \langle\bar{\Delta}\Delta_v^*\rangle\Delta_v(\hat{k}) - \langle\bar{\Delta}^*\Delta_v\rangle\Delta_v(\hat{k}) \\
&- \frac{1}{2}\Gamma\cos 2\delta_0 a_4(\langle|\bar{\Delta}|^2\rangle\bar{\Delta}(\hat{k}) + \langle\bar{\Delta}\Delta_v^*\rangle\Delta_v(\hat{k}) - \langle\bar{\Delta}^*\Delta_v\rangle\Delta_v(\hat{k})) \\
&+ \frac{\delta\tilde{g}}{g}\frac{a_1}{(\pi T_c)^2}(\langle|\bar{\Delta}|^2\rangle\bar{\Delta}(\hat{k}) - \langle\bar{\Delta}\Delta_v^*\rangle\Delta_v(\hat{k}) - \langle\bar{\Delta}^*\Delta_v\rangle\Delta_v(\hat{k})), \tag{21}
\end{aligned}$$

where

$$a_1(T) = 2\pi T \sum_{\omega>0} \frac{1}{\tilde{\omega}} = \ln\left(\frac{2\gamma}{\pi} \frac{\omega_c}{T}\right) + \psi(1/2) - \psi(1/2 + w), \tag{22}$$

$$a_3(T) = 2\pi T \sum_{\omega>0} \frac{1}{\tilde{\omega}^3} = -\frac{1}{2} \frac{1}{(2\pi T)^2} \psi^{(2)}(1/2 + w), \tag{23}$$

$$a_4(T) = 2\pi T \sum_{\omega>0} \frac{1}{\tilde{\omega}^4} = \frac{1}{6} \frac{1}{(2\pi T)^3} \psi^{(3)}(1/2 + w). \tag{24}$$

Finally, for the order parameter  $A_{\mu i}$  the following equation is obtained from (21):

$$\begin{aligned}
\alpha(T)A_{\mu i} + \frac{N_F}{15}a_3 \left\{ -\frac{1}{2}A_{\mu i}^*A_{vj}A_{vj} + \left[ \left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3}\right) + \delta_{sc} \right] A_{\mu i}A_{vj}^*A_{vj} \right. \\
\left. + A_{\mu j}A_{vj}A_{vi}^* + \left[ \left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3}\right) - \delta_{sc} \right] A_{\mu j}A_{vj}^*A_{vi} \right. \\
\left. - \left[ \left(1 - \frac{5}{6}\Gamma\cos 2\delta_0 \frac{a_4}{a_3}\right) + \delta_{sc} \right] A_{\mu j}^*A_{vj}A_{vi} \right\} = 0, \tag{25}
\end{aligned}$$

where  $\alpha(T) = \frac{1}{3}N_F[\ln \frac{T}{T_{c0}} + \psi(1/2 + w) - \psi(1/2)]$  and the strong-coupling contribution is described by a parameter

$$\delta_{sc} = \frac{5}{3} \frac{\delta\tilde{g}}{g^2} \frac{1}{a_3} \frac{1}{(\pi T_c)^2}. \tag{26}$$

In equations (25) and (26) the coefficients  $a_3$  and  $a_4$  are to be calculated at  $T = T_c$ .

Comparing equation (25) with its phenomenological Ginzburg–Landau counterpart:

$$\begin{aligned}
\alpha(T)A_{\mu i} + 2(\beta_1 A_{\mu i}^*A_{vj}A_{vj} + \beta_2 A_{\mu i}A_{vj}^*A_{vj}) \\
+ \beta_3 A_{\mu j}A_{vj}A_{vi}^* + \beta_4 A_{\mu j}A_{vj}^*A_{vi} + \beta_5 A_{\mu j}^*A_{vj}A_{vi} = 0, \tag{27}
\end{aligned}$$

the  $\beta$ -coefficients can be identified. Representing  $\beta_i$  as the sum of weak-coupling ( $\beta_i^{wc}$ ) and strong-coupling ( $\delta\beta_i^{sc}$ ) contributions, it is found that

$$-2\beta_1^{wc} = \beta_3^{wc} = 2\beta_{wc} = \frac{7\zeta(3)}{120} \frac{N_F}{(\pi T_c)^2} \frac{\psi^{(2)}(1/2 + w_c)}{\psi^{(2)}(1/2)}, \tag{28}$$

$$\beta_2^{wc} = \beta_4^{wc} = -\beta_5^{wc} = 2\beta_{wc} - \frac{\Gamma\cos 2\delta_0}{12^3} \frac{N_F}{(\pi T_c)^3} \psi^{(3)}(1/2 + w_c), \tag{29}$$

$$\delta\beta_1^{sc} = \delta\beta_3^{sc} = 0, \tag{30}$$

$$\delta\beta_2^{sc} = -\delta\beta_4^{sc} = -\delta\beta_5^{sc} = \delta\beta_{sc}, \tag{31}$$

$$\delta\beta_{sc} = \frac{1}{2(4g)^2} \left(\frac{\pi}{3}\right)^3 \frac{N_F}{(\pi T_c)^2} \left(\frac{IN_F}{1 - IN_F}\right)^2 \frac{1}{k_F\xi_c} \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)}, \tag{32}$$

where  $w_c = \frac{\Gamma}{2\pi T_c}$ . It can be easily verified that the weak-coupling coefficients  $\beta_i^{wc}$  reproduce the answer reported in [11].

In order to explore the domain of the phase diagram (in the Ginzburg–Landau region) where the B-phase overcomes the strong-coupling effects and is preferable as an equilibrium state in comparison to the A-phase (in zero magnetic field), we use a well known inequality

$$\beta_{12} + \frac{1}{3}\beta_{345} < \beta_{245}. \quad (33)$$

Introducing the normalized  $\beta$ -coefficients  $\bar{\beta}_i = \beta_i/|\beta_1^{wc}|$ , the criterion of thermodynamical stability of the B-phase in the Ginzburg–Landau region reads as

$$-2\delta\bar{\beta}_{345}^{sc} + 3\delta\bar{\beta}_{13}^{sc} < 1. \quad (34)$$

According to equation (28),  $|\beta_1^{wc}| = \beta_{wc}^0 R_{wc}$  where the ‘impurity’ renormalization factor for the weak-coupling coefficient  $\beta_{wc}$  is given by

$$R_{wc}(w_c) = \frac{\psi^{(2)}(1/2 + w_c)}{\psi^{(2)}(1/2)} \left( \frac{T_{c0}}{T_c} \right)^2. \quad (35)$$

On the other hand, following equation (32),  $\delta\beta_{sc} = \delta\beta_{sc}^0 R_{sc}$  with the ‘impurity’ renormalization factor

$$R_{sc}(w_c) = \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)} \frac{T_{c0}}{T_c}. \quad (36)$$

It is to be remembered that the ratio  $T_{c0}/T_c(w_c)$  is found from the Abrikosov–Gorkov-type equation

$$\ln(T_{c0}/T_c) + \psi(1/2) - \psi(1/2 + w_c) = 0. \quad (37)$$

The renormalization factors  $R_{wc}$  and  $R_{sc}$  are monotonically increasing functions of  $w_c$  although the strong-coupling effects are less susceptible to the quasiparticle scattering events.

Collecting these results, in the framework of the simple model adopted, the B-phase stability region near  $T_c$  is defined by the inequality

$$\delta\bar{\beta}_{sc} = \delta\bar{\beta}_{sc}^0(P)R(w_c) < \frac{1}{4}, \quad (38)$$

where the effects of the finite mean free path of the quasiparticles in the aerogel environment are accumulated in the renormalization factor

$$R(w_c) = \frac{R_{sc}(w_c)}{R_{wc}(w_c)} = a(w_c) \frac{T_c}{T_{c0}} \quad (39)$$

with

$$a(w_c) = \frac{\psi^{(1)}(1/2 + w_c)}{\psi^{(1)}(1/2)} \frac{\psi^{(2)}(1/2)}{\psi^{(2)}(1/2 + w_c)}. \quad (40)$$

It is to be noted that in equation (38) the presence of  $T_c/T_{c0}$  certainly stems from the fact that the strong-coupling corrections to the free energy contain extra powers in  $T_c/T_F$  (in comparison with the weak-coupling contribution). At the same time, equation (38) shows that  $R(w_c)$  is not simply equal to  $T_c/T_{c0}$  but contains an extra factor  $a(w_c)$  which increases with the quasiparticle scattering rate and competes with  $T_c/T_{c0}$  which decreases with  $w_c$ . The analysis shows that this competition favours  $T_c/T_{c0}$  and  $R_{wc} < 1$  at  $w_c > 0$ . In particular for the case with  $w_c \ll 1$ ,  $a(w_c) \simeq 1 + 2.37w_c$  and  $T_c/T_{c0} \simeq 1 - \frac{1}{2}\pi^2 w_c$ , so  $R(w_c) \simeq 1 - 2.56w_c$ .

Turning back to equation (37) it is concluded that, since in the quasiparticle scattering medium  $R(w_c) < 1$ , the condition of the stability of the B-phase in aerogel is less restrictive in comparison with the bulk case. This opens a way for the appearance of the B-like superfluid state in the pressure region  $P > P_{c0}$  which increases with the quasiparticle scattering intensity. As was mentioned in the introduction, in the case of 98% porosity aerogel the B-like phase near  $T_c$  and in zero magnetic field is observed up to the melting pressure  $P_m$ . For larger-porosity aerogel (with smaller  $w_c$ ), the PCP may appear in the pressure region  $P_{c0} < P < P_m$ , as mentioned in [13].

### 3. Conclusions

As is well known, an isotropic B-phase of superfluid  $^3\text{He}$  is stabilized at pressures below  $P_{c0} \simeq 21$  bar. At higher pressures the A-phase takes over due to the strong-coupling effects manifested as a feedback of the Cooper pairing on the quasiparticle attractive interaction.

On the other hand, in recent experimental studies (see [13]) of the phase diagram of a ‘dirty’ superfluid  $^3\text{He}$  confined to 98% porosity silica aerogel, it was established that a B-phase-like ordered state is stabilized at  $P > P_{c0}$  up to  $P = P_m$ . This observation indicates that the scattering of quasiparticles against the spatial irregularities of the porous medium modifies the free energy of superfluid  $^3\text{He}$  in favour of the B-phase as an equilibrium ordered state at high pressures. The free energy of the superfluid state can be viewed as containing the two contributions stemming from the weak-coupling and strong-coupling effects. The former contribution for the ‘dirty’ superfluid  $^3\text{He}$  has been investigated theoretically in [11] where it was shown that the ‘impurity’ renormalization of the weak-coupling part of the free energy (near  $T_c$ ) promotes the stabilization of the B-phase at pressures where in bulk superfluid  $^3\text{He}$  the A-phase is an equilibrium ordered state. This conclusion, as mentioned in [11], supposes that the strong-coupling effects are not more susceptible to the quasiparticle scattering than their weak-coupling counterpart. In using a simple model to treat the strong-coupling contribution to the free energy, we have demonstrated that the ‘impurity’ renormalization of this contribution is considerably weaker in comparison to the weak-coupling effects. This conclusion seems to be in accordance with the above-mentioned experimental observations.

### Acknowledgments

We are grateful to Professor Bill Halperin for informing us about the results described in [13]. This work was supported by the INTAS Grant N 97-1643 and the Georgian Academy of Sciences Grant N 2-19.

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